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Sovysko Artemii

(International School of Brno, Czech Republic)

APPLICATION OF COMPLEX NUMBERS IN ENGINEERING CALCULATIONS

E-mail: artemii.s@isob.cz

This study examines the use of complex numbers in calculations, knowledge of which is very useful for studying and better understanding processes in electrical and electronic engineering. Complex numbers are a universal tool in engineering, for example, in the transmission, transformation and use of energy in large electrical networks and the creation of various units with electrical circuits. The basis of this engineering lies in the properties of potential difference and electric current. Thus, the key factors of research, analysis and calculations depend on the approach:

– theoretical, from the point of view of physics, which involves understanding the processes in electrical circuits;

– practical, from the point of view of mathematics, which takes into account knowledge of complex numbers and the ability to solve complex mathematical problems.

Complex numbers are widely used in most industries, such as engineering physics, especially in computing and electrical operations.

The significance of the work lies in the simplification of calculations involving changes in resistance, current and voltage. For example, taking linear diagrams, this mathematical tool allows you to perform complex and multi-operational calculations, solving linear equations and exponential functions.

Complex number z is an element of the calculation system that goes beyond the real number and is represented by the sum of the real part **a** and imaginary part **b** as follows**:**

$$z = a + bi$$

An example of the use of complex numbers in electrical engineering is and represents impedance in alternating current circuits (Fig. 1):

$$|\mathbf{Z}| = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{R^2 + X^2}$$

where **R** means that the resistance is the real part on the complex plane because it represents the active power, and **X** means the net reactance, which is represented as the imaginary part on the complex plane [1].

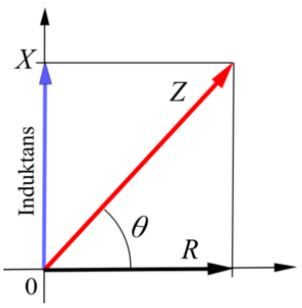

Fig 1. Impedance

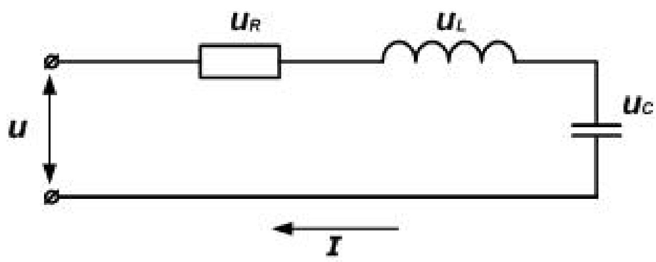

Fig. 2. Schematic diagram of the RLC series

An example of impedance calculation for a series RLC circuit (Fig. 2) [2]: a series circuit is given, consisting of a resistor (12 Om), an inductor (0.15 Hr), a capacitor (10 F) and an alternating voltage source (100 V, frequency 50 Hz).

Now, knowing the current, the potential difference across the capacitor, inductor and resistor we calculate the alternating current. Given the source voltage and impedance, Ohm's law can be applied, where impedance replaces resistance:

$$Z_{total} = \sqrt{R^2 + (X_L - X_C)^2}\, tan^{-1}(\frac{X_L - X_C}{R}) = tan^{-1}\left(\frac{15.29}{12}\right) =$$

$$= 19.44 < 51.87^0 \Omega$$

$$I = \frac{V_{source}}{Z_{total}} = \frac{100}{19.44} < (0^o - 51.87^o) = 5.14^o < 51.87^o$$

Now, knowing the current, the potential difference across the capacitor, inductor and resistor we get:

$$V_R = IZ_R = 5.14 < -51.87^o \times 12 = 61.7 < -51.87^o V$$
$$V_C = IZ_C = 5.14 < -51.87^o \times 31.83 < -90^o =$$
$$= 163.6 < [-51.87^o + (-90^o)] = 163.6 < -141.87^o V$$
$$V_L = IZ_L = 5.14 < -51.87^o \times 47.12 < 90^o = 163.6 < [-51.87^o + (90^o)]$$
$$= 242.2 < 38.13^o V.$$

The voltage across the inductor V gives exactly 180°, indicating that the capacitor and inductor are out of phase by π rad [4].

Phasors are complex representations of sinusoidal functions and are used to depict the relationship between multiple alternating voltages and currents with the same angular frequency **ω = 2π*f*** [5].

The calculation of sinusoidal currents and voltages involves complex differential equations common to circuit analysis that can be simplified to algebraic equations using vectors [6].

With the help of vector measurement units (PMU), the analysis and monitoring of the dynamics of the power system has become more efficient, which makes it possible to measure the phase angles between identical vectors in different places of a large power system, the entire network, including power plants and many consumers.

The sinusoidal signal has a formula and graphical interpretation (Fig. 3). The phase of a variable quantity at any moment in time can be represented by a vector diagram, which means they can be considered as "functions of time".

A complete sine wave can be represented by a single vector rotating at an angular velocity of **ω = 2π*f*,** where ***f*** is the frequency of the waveform.

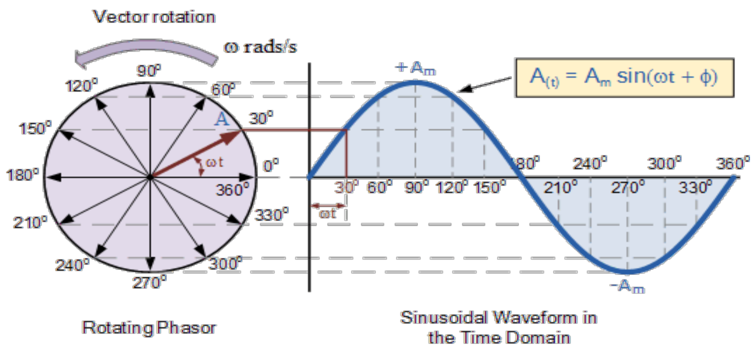

Fig. 3. Vector diagram [6]

A sine wave is a cosine wave shifted by - 90°. When working with sine or cosine waves at an angle, the following rules apply:

$$\sin(\omega t + \varphi) = \cos(\omega t + \varphi - 90^o)$$

When comparing two sine waves, it is most common to express their relationship as a sine or cosine with positive amplitudes, and this is achieved using the mathematical identities above.

Using the above relationships, we can convert any sine wave with or without angular difference or phase difference into a cosine wave or vice versa.

Adding or subtracting 180 degrees from the argument will change the negative amplitude to positive [6].

Determining the output signals and reactions of systems plays an important role in electronics, such as calculating current and voltage that vary over time. A tool such as Fourier series has been specially developed to work with periodic functions. The most prominent application of Fourier series is seen in the audio industry, where this mathematical tool is suitable for filtering and compressing signals. Noise canceling is present in many modern headphones and sound filters are possible thanks to the Fourier series algorithm embedded in them, which recognizes signal patterns and removes background noise or noise at certain places in the frequency of the sound wave. The series for the periodic function $\boldsymbol{f(t)}$ is determined by the formula:

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos\cos n\, \omega_0 t + b_n \sin\sin n\omega_0\, w\, t)$$

As a result of a series of transformations, we obtain a detailed formula for complex exponential Fourier series [9]:

$$f(t) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2T_0} \int_{-T_0}^{+T_0} f(t) e^{-jn\omega_0 t} \, dt \right) e^{-jn\omega_0 t} =$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} .$$

The output voltage function must be converted from general to Cartesian form; the complex conjugate solution will not change if the complex conjugate is taken instead.

Conclusion:

1. The fundamental use of this mathematical tool and the importance of complex numbers are shown using examples of calculations of impedance, vectors and Fourier series.

2. When performing calculations based on various properties of complex numbers, the transformation and simplification of equations is shown.

3. It has been confirmed [6] that the phasor is a quantity as a number that "has magnitude" and as a vector that "has direction".

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Velychkovych Andrii, Andrusyak Andriy, Petryk Ivan
(IFNTUOG, Ivano-Frankivsk)

THE METHOD FOR DETERMINING THE REACTION IN THE ADJUSTABLE SUPPORT OF AN OVERPASS OIL PIPELINE

E-mail: a_velychkovych@ukr.net

Consider the model of the two-section overpass of the oil pipeline in Fig. 1, which is built in a mountainous area. The overpass consists of the above-ground section *BC* with the length of $2l$, equipped with the adjustable support *K*, and the adjacent underground sections *AB* and *CD*, which rest on the soil base (placed on the bottom of the trench). The length of the underground sections of the oil pipeline is much longer than the above-